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RECIPROCAL SERVICE COST AND THE USE OF
CAPACITY

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Accountancy

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#595

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Summary:

This paper argues that for most planning purposes the costs of goods and services must reflect the costs of capacity as well as the variable costs of production. These costs are considered in a reciprocal service setting in which the capacities of the service departments are constrained. A linear programming model is used to formulate the simultaneous determination of the service costs including the costs of capacity. Finally, a suggestion is made that a full cost calculation of service costs may provide useful proxies of the opportunity costs of the internally generated service activities.

Reciprocal Service Costs and the Use of Capacity

A business, as it matures, is likely to accumulate an array of productive activities which were never contemplated when the company was founded. Many of these activities are added on a self-service basis as a matter of convenience, or they are added during periods of short supply, or to assure reliable quality, and for other reasons; such activities may well be continued provided that their full costs are adequately recognized and provided that management realizes what cost sacrifices are being made to achieve such convenience, etc.,... Their continuance or addition cannot be justified on the grounds that they need be charged with only part of the lists of costs normally charged to regular, principal operations. This is true because fixed costs commensurate with the activity will inevitably creep in to the operating costs structure.

"Creep," Robert L. Dixon, The Journal of Accountancy, July 1953, pp. 55.

The Discussion of Reciprocal Service Costs To Date

The problem of reciprocal service costs and the need that such costs be determined by the solution of a system of simultaneous equations was understood long before the widespread use of computers and the application of quantitative methods in managerial accounting [Newlove & Garner, 1949, T. Lang, 1944]. However, the practicable use of the simultaneous equation solution techniques was delayed until the development of computers capable of the necessary matrix operations [Manes, 1963], and shortly thereafter several papers introduced detailed explanations of the procedures involved in calculating reciprocal service center costs, notably Williams and Griffin [1964] and Churchill [1964].

The solutions suggested by these authors were based on matrices which expressed the related percentages of the total outputs of the interacting service departments consumed by other service departments, and their

approach shall be referred to as the "percentage of operations" method. Although this method calculates simultaneously determined, reciprocal service costs correctly, the firm must reformulate (and invert) the matrix of interdepartmental activities each and every time the relative proportions of the productive and service activities change.

Subsequently and separately, Ijiri [1968], Livingstone [1969] and Farag [1967, 1968] showed that the reciprocal service cost problem of the firm is directly comparable to the macroeconomic, input-output models developed by Leontief [1951], in which the submatrix of interacting service departments is like a subset of the national economy which produces only intermediate goods and services, and nothing for final consumer demand. This development of the topic, by its reference to the technological coefficients of production (which we shall refer to as the "input-output" approach) necessarily focused its attention on variable costs. The culminating piece of the "input-output" articles was R. Kaplan's [1973] paper. Following an earlier and similar demonstration by Livingstone [1968], Kaplan settled the confusion regarding methods of mathematically formulating the matrix of interdepartmental relationships, a confusion introduced by Manes [1965] and compounded by Minch and Petri [1972]. In so doing, Kaplan showed how the input-output approach was related to the earlier efforts of Williams and Griffin [1964] and others and presented a definitive explanation of the linear algebra operations required to calculate reciprocal or simultaneously determined, variable service department costs.¹

¹In a glow of well justified pride, Kaplan [1977] in his Beyer lecture at the University of Wisconsin states, "Perhaps the biggest triumph of mathematical modeling to cost accounting has been the matrix approach to allocating costs from interacting service departments to revenue producing departments."

In order to pursue the discussion and to illustrate points made, we shall utilize Kaplan's notation throughout and also we shall use the same problem chosen by him and later by Capettini and Salamon (C&S) [1977] to demonstrate a linear algebraic solution of reciprocal service costs. The problem which is attributed to David Green, is printed below as it appears in C. Horngren's Cost Accounting 4th Edition, p. 546.

The Prairie State Paper Company located a plant near one of its forests. At the time of construction, there were no utility companies equipped to provide this plant with water, power, or fuel. Therefore, included in the original facilities were (1) a water plant, which pumped water from a nearby lake and filtered it; (2) a coal-fired boiler room that produced steam, part of which was used for the manufacturing process and the balance for producing the electricity; and (3) an electric plant.

An analysis of these activities has revealed that 60 percent of the water is used for the production of steam and 40 percent is used in manufacturing. Half of the steam produced is used for the production of electric power and half for manufacturing. Twenty percent of the electric power is used by the water plant and 80 percent goes to manufacturing.

For the year 19__9, the costs charged to these departments were:

		Variable	Fixed	Total
(\$000's omitted)	Water Plant	\$ 2,000	\$ 8,000	\$ 10,000
	Steam Room	18,000	12,000	30,000
	Electric Plant	6,000	9,000	<u>15,000</u>
				\$55,000

Kaplan and C&S assume demand for the final product of 100,000 units and from this demand they generate a production schedule as follows:

(000's omitted)		Source			
User		x_1	x_2	x_3	x_4
Water	(x_1)	-	-	120	-
Steam	(x_2)	90	-	-	-
Electricity	(x_3)	-	90	-	-
Paper	(x_4)	60	90	480	-
<u>Totals</u>		150	180	600	100
		gals.	cubic ft.	K.W.h.	units

The "true" or simultaneously determined costs of water, steam and electricity respectively proceed from the solution of the following system:

$$X = V + AX ,$$

$$X - AX = V$$

$$X = (I-A)^{-1}V$$

where j^{th} term of $(m \times 1)$ vector X , represents the increase in total service departmental cost for production of one additional unit of service from the j^{th} department,

V is an $(m \times 1)$ vector, v_j is the traceable variable cost per unit of service department j , and

A is an $(m \times m)$ matrix, a_{ij} being the number of units of service department j required for each unit of output for service department i .

In this problem

$$A = \begin{bmatrix} 0 & 0 & .3 \\ .5 & 0 & 0 \\ 0 & .15 & 0 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} \$ 13.33 \\ \$100.00 \\ \$ 10.00 \end{bmatrix}$$

As solved correctly by Kaplan and C&S, per unit costs of water, steam and electricity are \$35.46, \$117.73, \$27.70 respectively for this system.

As elegant and as useful as the Kaplan article is in the development of the cost allocation methodology, it fails to consider a major part of

the above problem and of service department cost calculations in general, namely that of non-variable costs, that of costs of capacity. A central consideration, after all, in the creation and operation of many, if not most service departments, is the acquisition of a large piece of equipment, such as a power generating unit, a pumping system, a printing press, xerox machines or a computer; and the cost of computing services, for example, certainly should include the cost of buying or leasing the equipment and the expense of compensating the computer center supervision.

Ijiri, Livingstone and Farag at various points do show how matrices based on the Williams and Griffin "percentage of operations" approach can be used to determine full-cost, reciprocal service costs. And the C&S paper shows how avoidance of some semi variable overhead costs can be achieved by the correct computation of reciprocal service costs in a make or buy situation. However, most of the above cited literature omits discussion of fixed costs and no one has explicitly considered the reciprocal costs of a service department, the operations of which are approaching or have already reached capacity.

A Linear Programming Formulation of the Reciprocal
Service Cost Problem and the Costs of Capacity

We now take the same problem and reformulate it for the final output of 100 units used by Kaplan, C&S and for the additional datum of a contribution margin for paper of \$700 per unit of output. We can now formulate this problem as follows:²

²This approach was suggested to us by Professor John S. Hughes.

Problem #1

$$\begin{array}{ll}
 \text{Max} & -13.33 x_1 - 100 x_2 - 10 x_3 + 700 x_4 \\
 \text{S.T. (1)} & x_1 - .5 x_2 - .6 x_4 = 0 \\
 (2) & x_2 - .15 x_3 - .9 x_4 = 0 \\
 (3) & .8 x_1 + x_3 - 4.8 x_4 = 0 \\
 (4) & x_4 \leq 100
 \end{array}$$

and all x_j 's ≥ 0

The first three constraints are identities stating the respective technological relationships of x_1 , x_2 and x_3 to other service products and to final output, x_4 , and constraint (4) is the vector of final output demand [one product only for this problem]. In this form the expression of the problem is essentially definitional or even tautological since the convex set can be shown to be a point in 4 space. However, what we are interested in mostly in this formulation are the values of the shadow prices pertaining to constraints (1), (2), and (3).

As we already know with respect to the values of x_1 , x_2 , x_3 , the solution to the problem is:

<u>Output</u>	<u>Shadow Prices</u>
$x_1 = 150$	Constraint (1), Technological proportion = \$ 35.46
$x_2 = 130$	Constraint (2) " = 117.73
$x_3 = 600$	Constraint (3) " = 27.70
$x_4 = 100$	Constraint (4), Market Demand = 440.00

And Profit = 344,000

Very interestingly we note that the shadow prices for constraints (1), (2) and (3) are the vector of service department unit costs calculated by Kaplan and C&S. That this must be so can be seen by examination of the dual of Problem 1 [see Jensen, 1968, pp. 426-7].

$$\begin{array}{llll}
 \text{Min} & & & 100y_4 \\
 \text{S.T. (1)} & y_1 & - .8y_3 & \geq -13.33 \\
 & (2) & -.5y_1 + y_2 & \geq -100.00 \\
 & (3) & - .15y_2 + y_3 & \geq -10.00 \\
 & (4) & -.6y_1 - .9y_2 - 4.8y_3 + y_4 & \geq 700.00 \\
 & \text{and } y_4 \geq 0; & y_1, y_2, y_3 & \text{unrestricted.}
 \end{array}$$

The solution of the dual with respect to dual constraints (1), (2) and (3) is the solution of the simultaneous equation, reciprocal cost problem. This solution is not affected by the price of the firm's final product, i.e., the \$700 contribution margin of x_4 , nor is it in any way influenced by the demand vector of final output, i.e., 100 units of x_4 . But this is only true because we have assumed that there is unlimited capacity for all service required by final production. Is that a realistic assumption and what will happen to these reciprocal costs if any capacity limitations whatsoever are introduced?

Let us next consider problems 2-A, 2-B and 2-C in which, one by one, service department capacities are constrained in such a way that $x_1 \leq 140$, $x_2 \leq 175$ and $x_3 \leq 590$. Table 1 records the results of the solution of these three separate cases and those of a subsequent case to be considered later.

- - - - -
 Insert Table 1 about here
 - - - - -

Again shadow prices 1 through 3 represent the reciprocal service costs of the service center outputs. At first glance it is difficult to see how a per unit cost of \$35.46 for x_1 can soar to \$347.50 when x_1 capacity becomes binding. But the new shadow price can be shown to be the solution of

$$X^* = [I-A]^{-1} \cdot V^*$$

where

$$V^* = \begin{bmatrix} \$ 13.33 + 292.33 \\ 100.00 \\ 10.00 \end{bmatrix}$$

That is, the new shadow prices are the per unit costs of the three departments after the original traceable variable cost of x_1 , \$13.33, has been augmented by the shadow price of the capacity constraint for x_1 , \$293.33. Again this can be seen by examination of the dual of Problem 2-A:

Dual of Problem 2-A

$$\begin{array}{ll} \text{Min} & 100y_4 + 140y_5 \\ \text{S.T. (1)} & y_1 - .8y_3 + y_5 \geq -13.33 \\ & (2) \quad -.5y_1 + y_2 \geq -100.00 \\ & (3) \quad - .15y_2 + y_3 \geq -10.00 \\ & (4) \quad -.6y_1 - .9y_2 - 4.8y_3 + y_4 \geq 700.00 \\ & \text{and } y_4, y_5 \geq 0; y_1, y_2, y_3 \text{ unrestricted.} \end{array}$$

The interacting effect of x_1 on x_2 and x_3 can be traced to the increased costs of steam and electricity; for example, steam will cost its original cost of \$117.70 plus the increased cost of x_1 , [\$347.55 - \$35.46], times

.5, the coefficient a_{12} , which is the amount of units of water required to produce one unit of steam.

Here, an important point must be made. That is that the opportunity costs of all service departments rise sharply if any single one, or any combination of departments are at or near capacity. Under normal conditions, we can expect that in the well managed, going concern at least some service departments would be operating near or at full capacity. As Dixon pointed out, make or buy decisions which should rely solely on simultaneously determined variable costs will be very rare.

"If the company is in a position to make permanent additions to its side activities without adding to its fixed costs, something must be out of order. Fixed costs reflect capacity to operate (author's italics), and they appear throughout the organization. Evidently if fixed costs can be ignored in such a calculation, the company is overequipped and overstaffed for its regular work [pp. 49]."

R. Anthony, in a discussion of a well known managerial accounting case, Martall Blanket, points out that in a practical sense management should not think of shadow prices only when capacity is already completely occupied but instead should think in such terms whenever any major addition to activities would utilize remaining slack.³

³Anthony, R. A. and J. S. Reece, Managerial Accounting: Test and Cases, 5th Ed., [1975], In the solution to the Martall Blanket Case, p. 605, Anthony points out, "that a company does not have to be at full capacity before it switches from a contribution-analysis approach to a normal-pricing approach.... (I know of a steel company that sold its last increment of "excess" capacity to an auto producer at less than normal margin, only to have a nation-wide steel shortage occur months later. The result was a loss of tens of thousands of dollars...)".

A paradox: One particularly interesting result can be observed when one or more service departments are at capacity. Consider Problem 2-A, in which the water department is at full capacity. Without at least 150 units of water, x_1 , demand for 100 units of paper, x_4 , cannot be satisfied. The obvious alternatives facing the firm, if either alternative is available, is to add capacity to the constrained water department or to go outside and buy up to 10 units of water. Any acquisition of water at a cost of less than \$347.50 per unit will add to the total profit of the firm. However, a still better avenue of increasing profit may exist: e.g., to purchase steam! If additional water is not readily available, any price less than \$273.75 per unit for steam, will relieve the demand for water and increase profit. Problem 3 illustrates such a situation.

Problem 3

$$\begin{array}{ll}
 \text{Max} & -13.33 x_1 - 100 x_2 - 10 x_3 + 700 x_4 - 200 x_5 \\
 \text{S.T. (1)} & x_1 - .5 x_2 \quad 0 \quad - .6 x_4 \quad = 0 \\
 (2) & \quad x_2 - .15 x_3 - .9 x_4 + x_5 = 0 \\
 (3) & - .8 x_1 \quad \quad x_3 - 4.8 x_4 \quad = 0 \\
 (4) & \quad \quad \quad x_4 \quad \leq 100 \\
 (5) & x_1 \quad \leq 140 \\
 & \text{all } x_j \text{'s } \geq 0
 \end{array}$$

where x_5 is purchased steam and \$200 is its per unit cost.

The solution of Problem 3, presented in Table 1, increases profit by \$750. Depending on the relative prices of x_1 , x_2 and x_3 and the amounts used in their input-output relationship, it may be optimal to

buy more of a service which is already in excess capacity (180 units vs. 168 units) at a price higher than the simultaneously determined per unit cost of the purchased service (e.g., \$200.00 vs. \$117.73).

The Relationship of Opportunity Costs to
Fixed Costs and to Full Costing

Referring back to the example problem, we know that the profit that has been maximized is actually the direct costing, gross margin of the firm, i.e., that v_4 is the price less direct variable costs per unit of x_4 and that v_1 , v_2 , and v_3 are the traceable direct variable costs of service departments 1, 2, and 3 respectively. The price of paper is set high enough to cover all out-of-pocket fixed costs per period and some allocation of sunk fixed costs and also to yield a profit. The shadow prices of the capacity constraints, (which multiplied times positive-valued right hand side constraint value, b_i s equal profit), thus include fixed costs but they do so only in a very situation-specific way in that the fixed costs are reflected through the price of the paper and all fixed costs are charged to the service department whose capacity is binding. Supplemented by sensitivity analysis, shadow prices provide the necessary relevant data to the immediate short run, but they do not provide us with very useful calculations of reciprocal service costs for longer run planning.

There are three reasons for which the shadow prices related to the services department technological constraints may prove inadequate for furnishing useful costs for planning. The first, referred to in the previous paragraph is that they include an amount equal to the profit of the firm which uses the services. This is not the most serious

difficulty and it can be adjusted for by recalculating the optimal solutions of Problem 2-A, 2-B, and 2-C at a break even price for paper, \$550 per unit for 100 units of the final product.

As a result of a break even price for paper, that part of Table 1 which reported the service department costs augmented by capacity constraints to shadow prices has to be adjusted downwards as shown in Table 2.

- - - - -
Insert Table 2 about here
- - - - -

On this point Prof. Dixon argued that

"to justify the addition of a permanent new activity on the grounds of cost savings, the best available supplier's price must be shown not only to exceed the full cost of production, with no apportionable costs omitted, but it should be higher by an amount at least equal to the rate of profit which the company is able to make through its principal operation" (author's italics) (p. 53).

If one agrees with Dixon, no adjustment for profit would be necessary.

The second reason for being cautious in the use of shadow prices as costs is related to the mathematical nature of their calculation. For every constraint in the L.P. solution there is a basic variable and a shadow price; and for every non-slack, basic variable there is a positive shadow price. In the problem examined above, there are three kinds of constraints: 1) there are technological constraints, m in all, one for each service department; 2) there are m or less constraints corresponding to the capacities of the service departments; and 3) there are n or less market constraints for the n final products, the demand vector of final output. However there can only be $m + n$ or less non-slack basic variables with positive values and given the

reciprocal, input-output nature of the service departments, for any positive final output at all there will necessarily exist m technological constraint shadow prices. That means there remain only n shadow prices to be calculated for both the n or less marketing constraints and for the m or less service department capacity constraints.

As a result of these limits, under certain conditions it is possible for a service department capacity to be binding but to have a zero shadow price. In fact in parts 2-A, B and C, should two or more service departments ever operate at full capacity simultaneously in the production of paper, a degenerate solution results. Specifically, one or more of the slack variables, for constrained service department capacity remain in the basis at a zero value and, consequently their related shadow prices have zero values, all of the profit being imputed to the other service department via capacity constraints and/or market constraints. There being no shadow price for these service departments, no cost is attributed to the department in the solution of $X^* = [I - A]^{-1}V^*$. The resulting amounts calculated for reciprocal costs of service departments augmented for capacity costs can thus vary, depending on a tie-breaking rule in the L.P. algorithm rather than on the intrinsic costs of the service departments. In general, the problem of degeneracy and the resulting unsatisfactory conditions are more likely to result when m is larger than n .

The third reason for rejecting shadow prices for more than short run decision making purposes, i.e., for calculating standards or long run costs is also related to the nature of the mathematical solution of the L.P. problem. Shadow prices only appear when capacity is reached;

the shadow prices act like a cattleprod or electrified fence or an alarm at point of contact and not like a radar system which signals the approach of full capacity. Unlike the results of marginal analysis, in which solution values shift gradually for continuous functions, shadow prices change abruptly as solutions move from one extreme point on the convex set to another. For this same reason, shadow prices have not proved practical for transfer price determination [Manes, 1970]. And as Dopuch and Drake [1964] have pointed out the L.P. algorithm does not discriminate between marketing or capacity constraint.

And so, although we concur with Prof. Dixon's [1953, p. 50] position "that (capacity) fixed costs simply cannot be ignored in making the produce-or-purchase decision unless one is satisfied with a very short sighted analysis," for the above reasons shadow prices will often fail to make an appropriate provision for fixed costs.

Since we are not satisfied with shadow prices as service department costs, we are reduced to examining the full cost reciprocal cost. Resorting again to the paper company problem, we adjust the traceable cost vector V to include a "per unit" capacity cost. As Abel [1978] argues, capacity is created in anticipation of a certain volume of production and therefore the capacity costs should be allocated on the basis of planned utilization rather than on the basis of actual utilization.⁴

⁴We are mainly concerned with determination of long run costs for planning. Should the actual production deviate from the planned utilization volumes, variances will arise. Examination of the variances is also a managerial responsibility which is not discussed in this paper.

As Kaplan points out, the production schedule of service center output will flow from the firm's planned needs for final output ($Q = UP(I - A)^{-1}$ where U is the $(1 \times n)$ vector of demand for final product, P is the $(n \times m)$ matrix of input requirements of final production for service and Q is the $(1 \times m)$ vector of service department output required).

Thus for the problem at hand $Q = [150, 180, 600]$ and

$$V^* = \left[\begin{array}{c} \text{traceable} \\ \text{per unit} \\ \text{direct variable} \\ \text{cost} \end{array} + \frac{\text{traceable Fixed Cost}}{\text{Planned Capacity}} \right]$$

$$= \left[\begin{array}{c} \$13.33 + \frac{\$8,000}{150} \\ \$100.00 + \frac{\$12,000}{180} \\ \$10.00 + \frac{\$9,000}{600} \end{array} \right] = \left[\begin{array}{c} \$13.33 + \$53.33 \\ \$100.00 + \$66.67 \\ \$10.00 + \$15.00 \end{array} \right] = \left[\begin{array}{c} \$66.66 \\ \$166.67 \\ \$25.00 \end{array} \right]$$

Different V^* s and X^* s could be calculated for every change in the scheduled utilization of services but would probably only be calculated for major proposed changes in the use of capacity.

Solving for X^* , we obtain:

Service Department Costs per Unit

	Water	Steam	Electricity
Variable Cost	\$ 35.46	\$117.73	\$27.66
Capacity Cost	78.04	105.67	30.85
Full Cost	<u>\$113.50</u>	<u>\$223.40</u>	<u>\$58.51</u>

These full costs fall within the range of costs for 2-A, 2-B, and 2-C adjusted to omit profit (refer to Table 2). For the going concern, which sizes its service operations according to its needs, such costs

are far more representative of the opportunity costs of the firm than are a variable-cost-only computation.

One can argue then that, because of the inadequacies of shadow prices for any planning beyond the next immediate decisions, full costing is the best proxy of an opportunity cost system. Note that in a competitive economic system the outside supplier of services, in the long run, must recover his fixed or capacity costs as well. Therefore, the price demanded for services will include a provision for the capacity costs. For the purpose of long run planning, management must consider the efficiency of capacity utilization vis. a vis. others in the market. Often, the use of excess capacity to support new services leads to busy-ing facilities in activities they were not originally planned for. This view is also supportive of J. L. Zimmerman's [1979] recent paper which provides additional defense of traditional cost accounting techniques by showing how cost allocations 1) control the overconsumption of agent perquisites, 2) proxy the costs of degraded service, delays and future expansion, costs that arise when a common resource (or service) is shared by several decision makers. In that sense our paper is a formal expression of Zimmerman's second argument.

Conclusion

We prefaced this paper by an extended quotation from a paper written by Prof. R. L. Dixon twenty-six years ago. Current research often tends to set too low a value on, if not to disregard, the traditional wisdom of the past. On the topic of reciprocal service costs, and of the fixed costs related thereto, accounting research in the last 20 years has perhaps allowed the powerful conviction of marginal analysis and of crusading

efforts against the merit of any allocations--to blind it to the obvious, namely that the costs of goods and services must include their share of the cost capacity used in their production. We have showed how these costs are calculable when operations of the firms are placed in a mathematically programmable model. Finally, we have suggested that a full-cost calculation of reciprocal service costs provides useful and perhaps the best available proxies of the opportunity costs derived from the model.

Table 1

Reciprocal Costs Under Capacity Constraint*

	Prob. 1	Prob. 2-A $x_1 \leq 140$	Prob. 2-B $x_2 \leq 175$	Prob. 2-C $x_2 < 590$	Prob. 3 $x_1 \leq 140$ and x_5 Available at \$200 per unit
z (000's)	44,000	41,700	42,780	43,270	42,450
<u>Output (000's)</u>					
x_1	150	140	145.8	147.5	140
x_2	180	168	175.0	177.0	160
x_3	600	560	583.3	590.0	592
x_4	100	93.33	97.22	98.33	100
x_4 's Slack	-	6.67	2.78	1.67	-
x_5	-	-	-	-	18.8
<u>Shadow Prices</u>					
Tech Pro- portion x_1	\$ 35.46	\$347.5	\$ 66.66	\$ 97.87	\$ 200
Tech Pro- portion x_2	117.73	273.75	377.80	148.90	200
Tech Pro- portion x_3	27.76	51.06	66.67	105.70	40
Market Constraint x_4	440.00	-	-	-	208
Capacity Constraint x_1	-	293.3	-	-	154.70
Capacity Constraint x_2	-	-	244.4	-	-
Capacity Constraint x_3	-	-	-	73.33	-

*Note that, because of degeneracy in the solutions, only one capacity shadow price can be calculated even when 2 or 3 service departments are at full capacity. More on this point later.

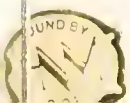
Table 2

Reciprocal Costs Under Capacity Constraint
at the Break Even Price

Shadow Prices	Prob 2-A $x_1 \leq 140$	Prob 2-B $x_2 \leq 175$	Prob 2-C $x_3 \leq 590$
Technological Proportion 1	(347.5) 241.10	(66.66) 56.00	(97.87) 76.57
Technological Proportion 2	(273.80) 220.60	(377.80) 289.10	(148.90) 138.30
Technological Proportion 3	(51.06) 43.09	(66.67) 53.37	(105.70) 79.08

The bracketed figures are the shadow prices including profit (as shown in Table 1) and the lower ones are the same figures with profit eliminated.





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